

NNLO Calculation of DIS; Precision Determination of the Strong Coupling Constant, Extraction of the Gluon Density, and Comments on “Hidden” Gluinos *

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Calculations of deep inelastic processes (DIS) to next-to-next to leading order are discussed. Fitting ep experiment in the range $2.5 \leq Q^2 \leq 230 \text{ GeV}^2$, we find the coupling $\alpha_s(M_Z^2) = 0.1163 \pm 0.0023$. We also get the gluon density $xG(x, Q^2 \simeq 10 \text{ GeV}^2) = 0.51 x^{-0.44} (1-x)^{8.1}$, and negative evidence for the existence of light gluinos

1. INTRODUCTION

Deep inelastic scattering (DIS), in particular of electrons/muons on protons, constituted one of the first probes of hadron structure. The calculation of QCD-induced scaling violations in the structure functions^[1] yielded some of the earliest checks of the quark-gluon theory of hadron interactions, as well as providing the first two loop determinations of the strong coupling constant^[2].

Let us set up some notation. Given the structure function $F_2(x, Q^2)$ in ep scattering, we split it into a singlet and a nonsinglet part,

$$F_2(x, Q^2) = F_{NS}(x, Q^2) + F_S(x, Q^2).$$

For the second we have to consider also the gluon structure function, $F_G(x, Q^2) = xG(x, Q^2)$, G the gluon density because they mix. We project the moments,

$$\mu_i(n, Q^2) = \int_0^1 dx x^{n-2} F_i(n, Q^2), \quad i = NS, S, G$$

For NS the QCD NNLO evolution equation is

$$\begin{aligned} \mu_{NS}(n, Q^2) = \mu_{NS}(n, Q_0^2) & \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{-\gamma_{NS}^{(0)}(n)/2\beta_0} \\ & \times \frac{1 + B_n^{(1)} \alpha_s(Q^2)/4\pi + B_n^{(2)} (\alpha_s(Q^2)/4\pi)^2}{1 + B_n^{(1)} \alpha_s(Q_0^2)/4\pi + B_n^{(2)} (\alpha_s(Q_0^2)/4\pi)^2}. \end{aligned}$$

The B may be written in terms of anomalous dimensions, $\gamma^{(N)}(n)$ and Wilson coefficients, $C^{(N)}(n)$ of order N : to NNLO we require $N = 0, 1, 2$. (For the singlet equations, see ref. 3.)

To compare the QCD predictions with experiment we need thus to evaluate $\gamma^{(N)}(n)$, $C^{(N)}(n)$

and invert the moments equations. This can be done with Altarelli–Parisi equations; but for this we would need the corresponding kernels, known only to NLO.^[4] It is also possible to invert the equations analytically if the analytic form of the $\gamma^{(N)}(n)$, $C^{(N)}(n)$ is known for all n (ref. 5 to LO, and ref. 2 to NLO). To NLO the calculations are complete^[2,4,6]; to NNLO we have the calculations of ref. 7 that provide us with the Wilson coefficients; but the $\gamma^{(N)}(n)$ are only known for a few values of n . Indeed, the calculations of Larin et al.^[8] give those corresponding to NS scattering for $n = 1$; and the singlet and nonsinglet $\gamma^{(N)}(n)$ in electroproduction for $n = 2, 4, 6, 8$. Therefore, before comparing with experiment some work has to be done. For the *nonsinglet* case, see ref. 9; we next briefly describe the method followed by us in the singlet case, ref. 3 to where we send for more details. We also present here, for the first time, the ensuing determination of the gluon density, as well as a few comments on the (negative) implications of our analysis for the existence of light gluinos.

2. BERNSTEIN AVERAGES

For a given value of Q^2 only a limited number of experimental points, covering a partial range of values of x , are available, so one cannot simply use the moments equations. A method devised to deal with such a situation is that of averages with the (modified) Bernstein polynomials (modified because, since only even moments are known, we have to consider polynomials in the variable x^2); for details on the method, see refs. 3, 10, and work quoted there. We define these polynomials

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for $k \leq n$ as

$$p_{nk}(x) = \frac{2\Gamma(n + \frac{3}{2})}{\Gamma(k + \frac{1}{2})\Gamma(n - k + 1)} x^{2k}(1 - x^2)^{n-k}.$$

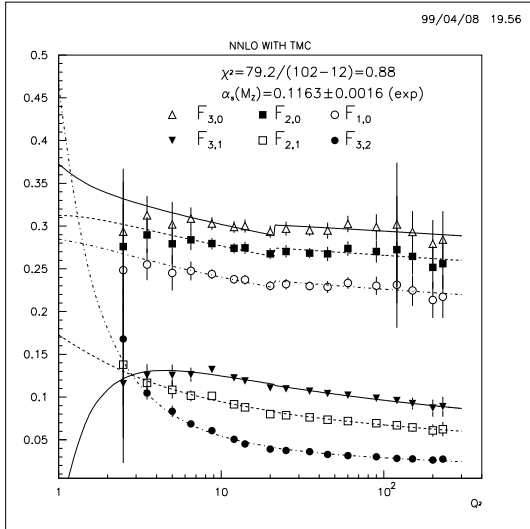
The $p_{nk}(x)$ are positive and have a single maximum located at $\bar{x}_{nk} \sim k/n$. They are concentrated around this point, with a spread, $\Delta x_{nk} \sim 1/n$ (accurate expressions for \bar{x}_{nk} , Δx_{nk} can be easily calculated, or looked for in ref. 3); and they are normalized to unity, $\int_0^1 dx p_{nk}(x) = 1$: so, the integral $\int_0^1 dx p_{nk}(x)\varphi(x)$ represents an average of the function $\varphi(x)$ in the region $\bar{x}_{nk} - \frac{1}{2}\Delta x_{nk} \lesssim x \lesssim \bar{x}_{nk} + \frac{1}{2}\Delta x_{nk}$. The values of the function $\varphi(x)$ outside this interval contribute little to the integral, as $p_{nk}(x)$ decreases to zero quickly there. Finally, using the binomial expansion it follows that the averages with the p_{nk} of a function can be obtained in terms of its even moments:

$$\begin{aligned} \varphi_{nk} &\equiv \int_0^1 dx p_{nk}(x)\varphi(x) = \\ &= \frac{2(n-k)!\Gamma(n + \frac{3}{2})}{\Gamma(k + \frac{1}{2})\Gamma(n - k + 1)} \sum_{l=0}^{n-k} \frac{(-1)^l}{l!(n-k-l)!} \mu_{2k+2l}^\varphi, \end{aligned}$$

and $\mu_{2l}^\varphi = \int_0^1 dx x^{2l}\varphi(x)$. We will thus consider our experimental input to be given by averages

$$F_{nk}^{(\text{exp})}(Q^2) \equiv \int_0^1 dx p_{nk}(x) F_2^{(\text{exp})}(x, Q^2), \quad (2.1)$$

$F_2^{(\text{exp})}$ the experimental structure function.^[11] The experimental points (and the theoretical fit) are shown in the figure.



3. NUMEROLOGY

We present in Table 1 a compilation of the results obtained with our calculations at LO, NLO and NNLO, with TMCs (target mass corrections) taken into account; the fit to the data itself is shown, for the NNLO calculation (with TMC) in the previous figure. Note the (small) jumps in the theoretical curves at the location of the mass threshold, $Q^2 = m_b^2$; they occur because we have joined the theoretical formulas from $n_f = 4$ to $n_f = 5$ at that point, using the method of ref. 12, which is not exact.

| Order | $\Lambda(n_f = 4)$ | $\alpha_s(M_Z^2)$ | $\chi^2/\text{d.o.f.}$ |
|-------|--------------------|---------------------|------------------------|
| LO | 215 ± 73 | 0.135 ± 0.007 | $212/102 - 12$ |
| NLO | 282 ± 40 | 0.1175 ± 0.0027 | $80.0/102 - 12$ |
| NNLO | 283 ± 25 | 0.1163 ± 0.0016 | $79.2/102 - 12$ |

Table 1

We have 12 parameters: the twelve moments at the initial value, $\mu_i(n, Q_0^2)$, $n = 2, 4, 6, 8$ and $i = S, G, NS$, minus one moment, $\mu_G(2, Q_0^2)$ which is deduced from $\mu_S(2, Q_0^2)$ via the momentum sum rule; plus the QCD coupling parameter, Λ . The initial value is taken $Q_0^2 = 8.75 \text{ GeV}^2$, and we evolve to all other values of Q^2 , in the range $2.5 \text{ GeV}^2 \leq Q^2 \leq 230 \text{ GeV}^2$, with the QCD evolution equations; then construct the combinations entering the Bernstein averages, and fit the values of these obtained from experiment. In Table 1, only experimental (*statistical*) errors of the fit are shown; systematic (*theoretical*) errors will be discussed below. The NLO corrections are clearly seen in the fit: the $\chi^2/\text{d.o.f.}$ decreases from a largish value of ~ 2.4 to a very good ~ 0.89 . The fit is so good at this order that there is little room for improvement when going to NNLO; nevertheless, an improvement is seen. Not in the $\chi^2/\text{d.o.f.}$; but including NNLO corrections leads to a noticeable gain both in the quality of the determination of the coupling, and in the stability of the fits.

Estimated systematic errors, originating from various sources, are shown for the NNLO calculation in Table 2. No TMC means that we have neglect target mass corrections. The corresponding error is *not* included when evaluating the overall theoretical error, since we *do* take into account TMCs in our central value. “Interp.” refers to the *theoretical* errors inherent in the calculation of

the integrals in (2.1) giving the experimental averages which arise because, for this calculation, it is necessary to interpolate the experimental points. We have used two different interpolation methods, one assuming a hard Pomeron (refs. 13) but with independent fits for every Q^2 , which furnishes our central value, or using the MSRT98,^[14] that gives the estimated error. HT means that we have taken into account higher twists phenomenologically, by adding, to $\mu_{NS}(n, Q^2)$, the correction $\mu_{NS}^{HT}(n, Q^2) = n(a\Lambda^2/Q^2)\mu_{NS}(n, Q^2)$. a is free parameter whose fitted value is $a = -0.202 \pm 0.030$, a very reasonable number. “Quark mass effect” means that we cut off the b quark threshold region, instead of matching through quark thresholds; the variation in Table 2 takes into account also the variations due to the uncertainty in the m_b mass. Q_0^2 to 12 GeV² means that we take the input moments defined at this value of the momentum, $\mu_i(n, Q_0^2 = 12 \text{ GeV}^2)$, $i = S, G, NS$ instead of $Q_0^2 = 8.75 \text{ GeV}^2$ as was done to obtain the results of Table 1. Finally, NNNLO is the estimated effect of the (likely) larger sources of corrections of higher order in α_s .

| Source of error | $\Lambda(n_f = 4, 3 \text{ loop})$ | $\Delta\Lambda$ | $\Delta\alpha_s(M_Z^2)$ |
|--------------------------------|------------------------------------|-----------------|-------------------------|
| No TMC | 292 | 9 | 0.0006 |
| Interp. | 273 | 10 | 0.0007 |
| HT | 292 | 9 | 0.0006 |
| Quark mass effect | 299 | 16 | 0.0010 |
| Q_0^2 to 12 GeV ² | 294 | 11 | 0.0007 |
| NNNLO | 289 | 6 | 0.0004 |

Table 2

Composing quadratically *all* errors we find

$$\begin{aligned} \Lambda(n_f = 4, 3 \text{ loop}) &= 283 \pm 25 \text{ (stat.)} \pm 24 \text{ (syst.)} \\ &= 283 \pm 35 \text{ MeV;} \\ \alpha_s^{(3\text{loop})}(M_Z) &= 0.1163 \pm 0.0023. \end{aligned}$$

In Table 3 we compare our results to previous determinations^[15] for $\alpha_s(M_Z^2)$, to the NNLO level (but excluding e^+e^- annihilations.) DIS means deep inelastic scattering, Bj stands for the Bjorken, GLS for the Gross–Llewellyn Smith sum rules. The xF_3 result is that of ref. 9.

The previously existing average, also taking into account NLO calculations, was $\alpha_s(M_Z^2) =$

0.118 ± 0.006 ; when including both our result and that of ref. 9 the new average and error become

$$\alpha_s(M_Z^2) = 0.1165 \pm 0.0016.$$

We discuss briefly how the calculation could improve. Adding the values of $\gamma(n)$ for a few more values of n would allow us to extend the range and increase the precision of our evaluations: already two more moments would probably decrease the error in Λ in some 30%. This is a difficult task, but it appears that Moch and Vermaseren are on the way to it (see the paper by S. O. Moch, these Proceedings; it is even possible that the analytic expression be found for $\gamma(n)$).

| Process | Average Q^2 or Q^2 range (GeV) | $\alpha_s(M_Z^2)$ |
|--------------------------------|------------------------------------|------------------------------|
| τ decays | 1 – (1.777) ² | 0.119 |
| $Z \rightarrow \text{hadrons}$ | (91.2) ² | 0.124 |
| DIS; ν , Bj | 2.5 | $0.122^{+0.005}_{-0.009}$ |
| DIS; ν , GLS | 3 | 0.115 ± 0.006 |
| DIS; ν , xF_3 | 5 – 100 | 0.117 ± 0.010 |
| DIS; $e/\mu p$, F_2 | 2.5 – 230 | 0.1163 ± 0.0023 |
| Average DIS | 2.5 – 230 | $0.1168^{+0.0019}_{-0.0020}$ |

Table 3

4. THE GLUON STRUCTURE FUNCTION

A spin-off from our results is that we also get moments of the gluon structure function, in particular at our starting value of $Q^2 = Q_0^2$:

$$\begin{array}{ll} Q_0^2 = 8.75 & Q_0^2 = 12 \\ \mu_G(2) = 0.242041 \pm 0.003 & 0.2390 \pm 0.0015 \\ \mu_G(4) = 0.0020 \pm 0.00023 & 0.0020 \pm 0.0055 \\ \mu_G(6) = 0.00119 \pm 0.00050 & 0.00142 \pm 0.0013 \\ \mu_G(8) = (0.99 \pm 0.86)10^{-3} & (1.00 \pm 0.84)10^{-3}. \end{array}$$

The results, fairly stable, do *not* fix the gluon density; to get it, extra assumptions have to be made. Following the hard Pomeron model we take the functional form

$$F_G(x, Q^2) \equiv xG(x, Q^2) = A_G x^{-\lambda} (1-x)^\nu,$$

fixing $\lambda = 0.44$ (ref. 13) to avoid spurious minima. Fitting the moments gives

| | | |
|----------------------------|--------------------------|--------------------|
| $Q^2 = 8.75 \text{ GeV}^2$ | $Q^2 = 12 \text{ GeV}^2$ | $Q^2 = 8.75 \& 12$ |
| $A_G = 0.52$ | $A_G = 0.51$ | $A_G = 0.51$ |
| $\nu = 8.17$ | $\nu = 8.08$ | $\nu = 8.11$ |
| $\chi^2 = 5.8$ | $\chi^2 = 2.4$ | $\chi^2 = 9.0$ |

the last if making a global fit. The value of the $\chi^2/\text{d.o.f.} = 9.0/(8-2)$ in this last case is quite reasonable: note that A_G and ν should depend on Q^2 , so we are, in the last fit, obtaining an average.

It is interesting to compare the value obtained for A_G with the prediction of the hard Pomeron model. We write^[13]

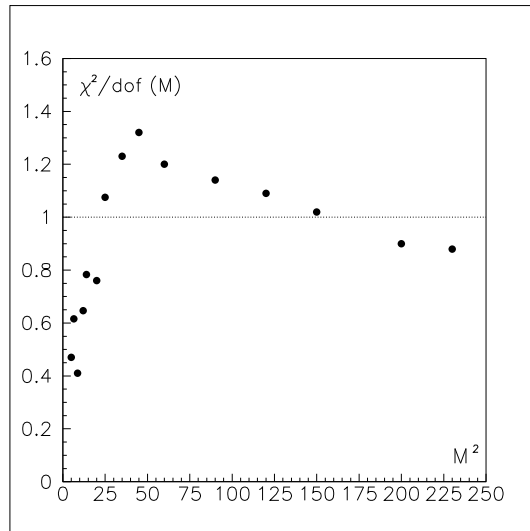
$$F_S(x, Q^2) \underset{x \rightarrow 0}{\simeq} \langle e_q^2 \rangle A_S(Q^2) x^{-\lambda}, \quad \lambda = 0.44,$$

$$A_S(Q^2) = \langle e_q^2 \rangle \alpha_s(Q^2)^{-d_+(1+\lambda)} B_S, \quad B_S = \text{const.},$$

$A_G/B_S = [d_+(1+\lambda) - D_{11}(1+\lambda)]/D_{12}(1+\lambda) \simeq 4.82$. Thus $\langle e_q^2 \rangle B_S \simeq 2.1 \times 10^{-3}$, in the ballpark of the values found in ref. 13.

5. COMMENTS ON “HIDDEN” GLUINOS

It is recurrently suggested that “hidden” light gluinos could exist.^[16] They would alter the evolution of α_s : in this respect, the agreement of our determinations with those made at smaller (τ decay) or higher energy (Z decay) provides strong evidence against their existence. Direct negative evidence is obtained as follows. Let $\chi^2/\text{d.o.f.}(M)$ be the chi square per degree of freedom, taking into account *only* experimental points with $Q^2 \leq M^2$. If, at a given $M = M_0$ a channel for the production of particles not taken into account in the analysis opened, then $\chi^2/\text{d.o.f.}(M)$ should jump at $M = M_0$, and would continue deteriorating for larger M , as we are fitting with *wrong* theoretical formulas: neglect of NLO corrections deteriorates the $\chi^2/\text{d.o.f.}$ substantially, and gluinos contribute already to LO. A plot of $\chi^2/\text{d.o.f.}(M)$ at different M is given in the figure (the full dots; the straight line is the ideal value $\chi^2/\text{d.o.f.}(M) = 1$). The increase of the $\chi^2/\text{d.o.f.}(M)$ near $M = m_b$ is clearly seen, due to our approximate treatment of the threshold; but, since we are using correct theoretical formulas (that is, with $n_f = 5$) above that value, $\chi^2/\text{d.o.f.}(M)$ decreases as we get far from m_b . This shows the effectiveness of the method.



No other step, or deterioration is seen in the whole range. Now, the presence of a particle with mass M_0 produces effects in the Bernstein averages below $Q^2 \simeq M_0^2$ because they involve integrals with the variable x . We are sensitive to energies up to $E_{\text{Max.}} \simeq \sqrt{(1-\bar{x})Q_{\text{Max.}}^2/\bar{x}} \simeq 30 \text{ GeV}$, and slightly above. So it follows that we can exclude gluinos with masses below $16 \sim 20 \text{ GeV}$.

REFERENCES

- 1 D. J. GROSS AND F. WILCZEK, *Phys. Rev.*, **D9** (1974) 980; H. GEORGI AND H. D. POLITZER, *Phys. Rev.*, **D9** (1974) 416; I. HINCHLIFFE AND C. H. LLEWELLYN SMITH, *Nucl. Phys.*, **B128** (1977) 93; A. DE RÚJULA, H. GEORGI AND H. D. POLITZER, *Ann. Phys. (NY)*, **103** (1977) 315.
- 2 A. GONZÁLEZ-ARROYO, C. LÓPEZ AND F. J. YNDURÁIN, *Nucl. Phys.*, **B153** (1979) 161; **B159** (1979) 512; **B174**, (1980) 474.
- 3 J. SANTIAGO AND F. J. YNDURÁIN, FTUAM 99-8/UG-FT-97/99 (hep-ph/9904344).
- 4 G. CURCI, W. FURMANSKI AND R. PETRONZIO, *Nucl. Phys.*, **B175** (1980) 27; W. FURMANSKI AND R. PETRONZIO, *Phys. Lett.*, **97B** (1980) 437.
- 5 D. J. GROSS, *Phys. Rev. Lett.*, **32** (1974) 1071.
- 6 E. G. FLORATOS, D. A. ROSS AND C. T. SACHRAJDA, *Nucl. Phys.*, **B129** (1978) 66; (E) **B139** (1978) 545; W. A. BARDEEN ET AL., *Phys. Rev.*, **D18** (1978) 3998.
- 7 W. L. VAN NEERVEN AND E. B. ZIJLSTRA, *Phys. Lett.*, **B272** (1991) 127 and 476; *ibid* B273 (1991) 476; *Nucl. Phys.* **B383** (1992) 525.
- 8 S. A. LARIN ET AL., *Nucl. Phys.*, **B427** (1994) 41 and **B492** (1997), 338.
- 9 A. L. KATAEV, G. PARENTE AND A. V. SIDOROV, IC/99/51 (hep-ph/9905310).

- 10 F. J. YNDURÁIN, *Phys. Lett.*, **74B** (1978) 68; see also J. WOSIEK AND K. ZALEWSKI, *Acta. Phys. Pol.*, **B11** (1980) 697.
- 11 L. W. WHITLOW ET AL., *Phys. Lett.*, **B282** (1992) 475; A. BENVENUTI ET AL., *Phys. Lett.*, **B223** (1989) 485; M. R. ADAMS ET AL., *Phys. Rev.*, **D54** (1996) 3006; M. DERRICK ET AL., *Z. Phys.*, **C72** (1996) 399 and S. AID ET AL., *Nucl. Phys.*, **B470** (1996) 3.
- 12 K. G. CHETYRKIN ET AL., *Phys. Rev. Lett.*, **74** (1995) 2184.
- 13 C. LÓPEZ AND F. J. YNDURÁIN, *Nucl. Phys.*, **B181** (1980) 231; *ibid.*, **B187** (1981) 157; K. ADEL, F. BARREIRO AND F. J. YNDURÁIN, *Nucl. Phys.*, **B495** (1997) 221.
- 14 A. MARTIN ET AL., *Eur. Phys. J.*, **C4** (1998) 463.
- 15 S. BETHKE, *Nucl. Phys. Suppl.*, **B64** (1998) 54.
- 16 G. FARRAR, *Nucl. Phys. Suppl.*, **B62** (1998) 485.